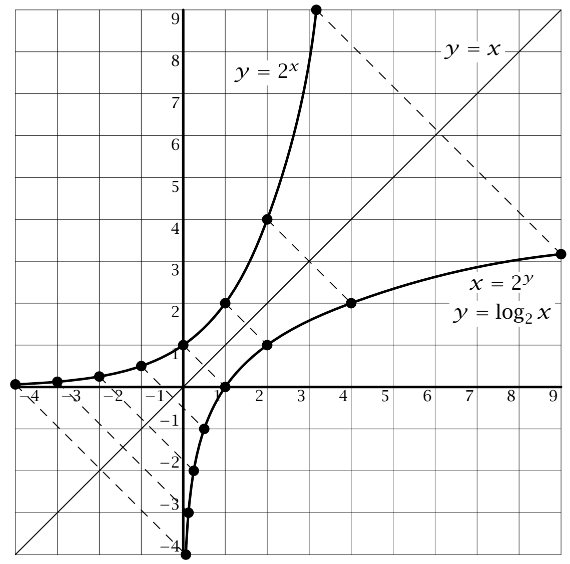
Summary of Exponential and Logarithmic Functions

The exponential function $y = a^x$, where a is positive number not equal to 1, has ***domain*** all x and ***range*** all $y > 0$. It's graph passes both the Vertical Line Test (since it's already a function) and the Horizontal Line Test (since it's increasing). If the graph of a function passes the HLT, then it is one-to-one. If the graph of $y = a^x$ is reflected across the line $y = x$ we obtain the graph of the equation $x = a^y$ by interchanging x and y. The graph of $x = a^y$ passes the VLT since the graph of $y = a^x$ passes the HLT. We write this new function as $y = \log_a x$ instead of $x = a^y$. Since everything gets interchanged, the logarithmic function $y = \log_a x$ has ***domain*** all $x > 0$ and ***range*** all y. This new function is the ***inverse*** of the exponential function. Two functions are inverses of each other if and only if they undo each other.



	Exponential	***Logarithmic***
1. Definition	$y = a^x$	$\log_a y = x$
2. Definition	$a^y = x$	$y = \log_a x$
3. Exponent= 0	$a^0 = 1$	$\log_a 1 = 0$
4. Exponent= 1	$a^1 = a$	$\log_a a = 1$
5. Product Property	$a^r a^s = a^{r+s}$	$\log_a(MN) = \log_a M + \log_a N$
6. Quotient Property	$\dfrac{a^r}{a^s} = a^{r-s}$	$\log_a\left(\dfrac{M}{N}\right) = \log_a M - \log_a N$
7. Power Property	$(a^r)^p = a^{rp}$	$\log_a M^p = p\log_a M$
8. One-to-One Property	$a^{x_1} = a^{x_2} \Leftrightarrow x_1 = x_2$	$\log_a x_1 = \log_a x_2 \Leftrightarrow x_1 = x_2$
9. Inverse Property	$a^{\log_a x} = x$ for $x > 0$	$\log_a a^x = x$ for any x
10. "$b2a$" Base Change Formula		$\log_b M = \dfrac{\log_a M}{\log_a b}$
11. Common Logarithm (base 10)		$\log x$ (no base means base 10)
12. Natural Logarithm (base e) $e \approx 2.7.18281828\ldots$		$\ln x$

FACTS: • The three bases of logarithms in use today are 10, e, and 2.
• Our number system is base 10, even though base 12 is much, much better.
• Base e occurs throughout mathematics, whether pure, applied, or financial, and that is why $\ln x$ is called the *natural* logarithm. If you take calculus, statistics, or finance you'll see e all the time.
• Base 2 logarithm occurs in computer science and information theory since the number of bits N of information is a power of 2, the exponent, $\log_2 N$, is a measure of the amount of information.
• The two most natural bases in the universe are 2 and e. Base 10 is useful to us only because we have a number system based on the biological accident of our having ten fingers. Base 10 is used in Chemistry - pH is an example. An octopus with $8 = 2^3$ tentacles probably thinks in base 2 (or 8, 16, or better still, 12) and uses natural logarithms, but couldn't care less about base 10!